CHAPTER 2 - BASIC LAWS

List of topics for this chapter :
Ohm's Law
Nodes, Branches, and Loops
Kirchoff's Laws
Series and Parallel Resistors
Wye-Delta Transformations
Applications

OHM'S LAW

Ohm's law states that the voltage across a resistor is directly proportional to the current flowing through the resistor.

$$v = i\,R$$

Problem 2.1   Given an incandescent light bulb rated at 75 watts and 120 volts, find the "hot" resistance and "cold" resistance of the light bulb.

An incandescent light bulb is the most common source of light produced by electrical energy. A current is made to flow through a wire, called a filament. Since this wire has resistance, the wire absorbs power, so much so that it "glows" and gives off light. Ratings on these types of light bulbs are for steady-state operating conditions. So, we have

$$I = \frac{P}{V} = \frac{75}{120} = \frac{25}{40} \text{ amps}$$

$$R = \frac{V}{I} = \frac{120}{25/40} = 192 \text{ ohms}$$

Hence, **the "hot" resistance is 192 ohms**.

Most wire has what we call a positive thermal resistance characteristic; that is, as the wire temperature increases, the resistance increases.

It is sufficient to say at this time that **the "cold" resistance is less than 192 ohms**.

Problem 2.2   Solve for v, i, and R for the following :
(a) $R = 10$ ohms and $i = 2$ amps
(b) $R = 5$ ohms and $v = 20$ volts
(c) $v = 10$ volts and $i = 5$ amps
(d) $i = 3$ amps and $v = 20$ volts

(a)   v = **20 volts**

(b)   i = **4 amps**

(c)   R = **2 ohms**

(d)   R = **6.667 ohms**

NODES, BRANCHES, AND LOOPS

A branch represents a single element, such as a voltage source or a resistor. A node is the point of connection between two or more branches. A loop is any closed path in a circuit.
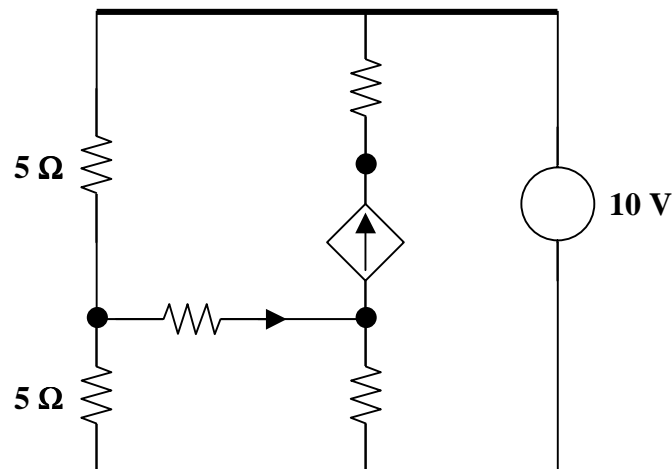
Problem 2.3   *[2.7]*   Determine the number of branches and nodes in the circuit in Figure 2.1.
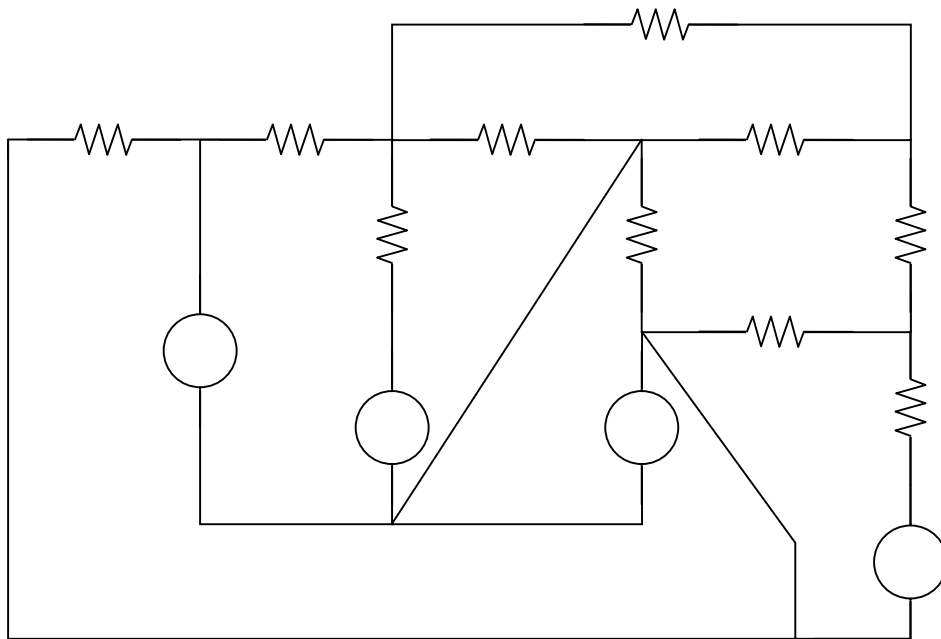

Figure 2.1

There are 7 elements, 1 dependent current source, 1 independent voltage source, and 5 resistors, which implies that there are **7 branches**. There are **5 nodes** as indicated by the dark circles and dark lines in the circuit below.
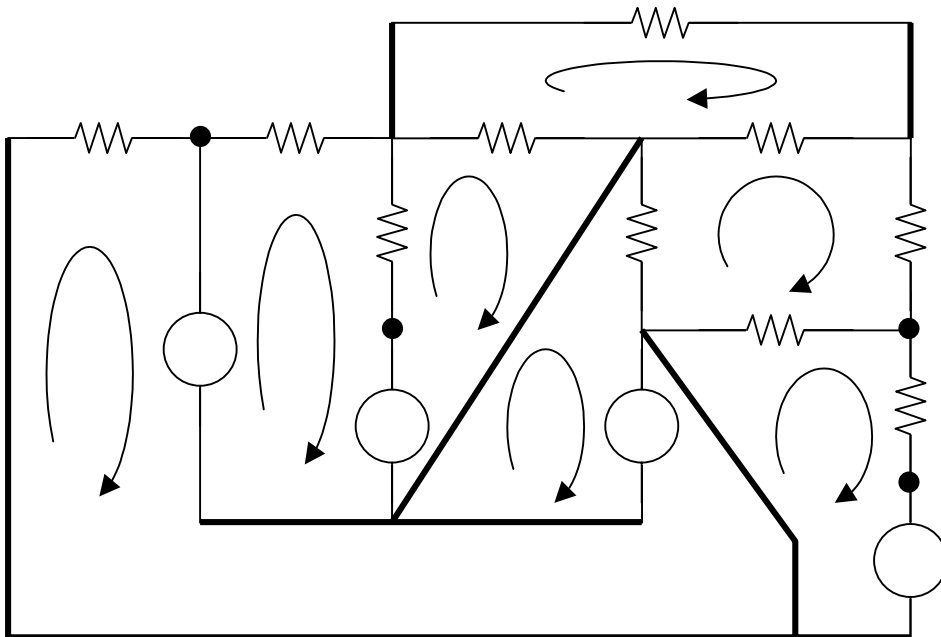
Problem 2.4   Identify all the nodes, branches, and independent loops in Figure 2.1.

Figure 2.1

There are **8 nodes**, as indicated by the dark circles and dark lines in the circuit below. There are **14 branches**, 4 independent voltage sources and 10 resistors. There are **7 independent loops**.
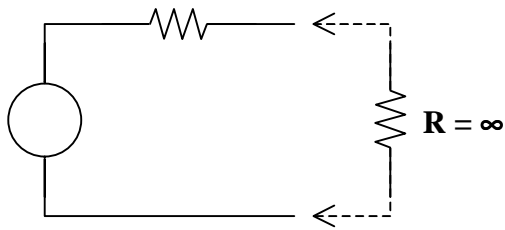

Check.   Does this satisfy the fundamental theorem of network topology?
$$b = l + n - 1 = 7 + 8 - 1 = 14 \qquad \text{YES!}$$

Problem 2.5   In **Figure 2.0**, is there a loop? How many nodes are there?

Figure 2.0

A loop is any *closed* path; therefore it is easy to say there is no loop. However, there really is a loop since there is an infinite resistance connecting the end terminals together. Thus, **there is one loop**. There are **three nodes**, one where the voltage source and resistor join and the two at the output terminals. In addition, there are **three branches**. The voltage source, the resistor, and the infinite resistance. Thus, the fundamental theorem of network topology is satisfied.

Problem 2.6
(a)   In a circuit containing 26 branches and 12 nodes, how many independent loops will satisfy the fundamental theorem of network topology?
(b)   In a circuit with 22 branches, is it possible to have 28 nodes?

$$b = l + n - 1 \quad \textit{\{the fundamental theorem of network topology\}}$$

(a) $26 = l + 12 - 1$
         $l = 26 - 12 + 1$
         $l = 15$
         A circuit with 26 branches and 12 nodes will have **15 independent loops**.

(b) $22 = l + 28 - 1$
         $l = 22 - 28 + 1$
         $l = -5$
         No. It is not possible to have a circuit with 22 branches and 28 nodes because a circuit cannot have –5 loops.

KIRCHOFF'S LAWS

Kirchoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero. Equivalently,
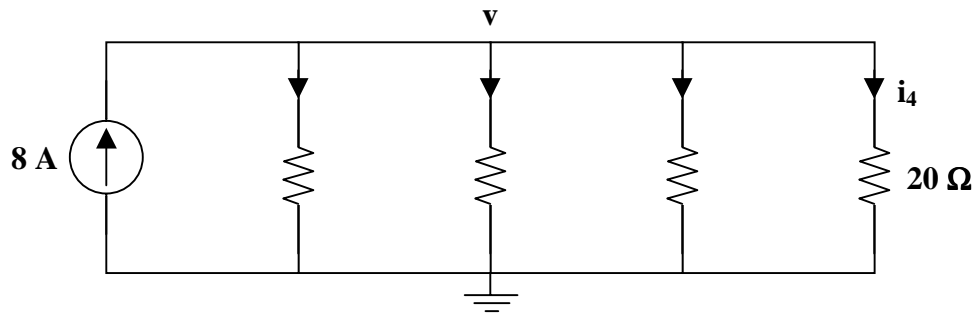
the sum of the currents entering a node = the sum of the currents leaving the node

Kirchoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero. Equivalently,

the sum of the voltage drops around a loop = the sum of the voltage rises around the loop

Problem 2.7 Determine all the currents and voltages in Figure 2.1.

Figure 2.1

➤ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➤ **PRESENT everything you know about the problem.**
The goal of the problem is to determine v, i_1, i_2, i_3, and i_4.

➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
KCL, involving currents entering and leaving a node, is the obvious circuit analysis technique to use, especially since the circuit has only one node. KVL could be used but would be more complicated due to the fact that there are five loops in the circuit. Using KVL creates four equations with four unknowns (since the fifth loop current is the current source), compared to one equation and four unknowns using KCL.

With one equation and four unknowns, constraint equations must be found. Realizing that v is the voltage across each component is the key. Ohm's law will be used to determine the relationship between the voltage and each of the four unknown currents. Using substitution, an equation in terms of v can be found and solved.

➤ **ATTEMPT a problem solution.**

KCL : $\qquad 8 = i_1 + i_2 + i_3 + i_4$

Ohm's law : $\quad v = 20i_1 = 10i_2 = 5i_3 = 20i_4$

$\qquad\qquad i_1 = v/20 \qquad i_2 = v/10 \qquad i_3 = v/5 \qquad i_4 = v/20$

Substitute the current equations found using Ohm's law into the equation found using KCL.
Then, solve for v.

$$8 = v/20 + v/10 + v/5 + v/20$$
$$160 = v + 2v + 4v + v$$
$$160 = 8v$$
$$v = 20 \text{ volts}$$

Hence,

$$i_1 = v/20 = 20/20 = 1 \text{ amp} \qquad\qquad i_2 = v/10 = 20/10 = 2 \text{ amps}$$
$$i_3 = v/5 = 20/5 = 4 \text{ amps} \qquad\qquad i_4 = v/20 = 20/20 = 1 \text{ amp}$$

➤ **EVALUATE the solution and check for accuracy.**
   Check the node equation found using KCL,
$$8 = i_1 + i_2 + i_3 + i_4$$
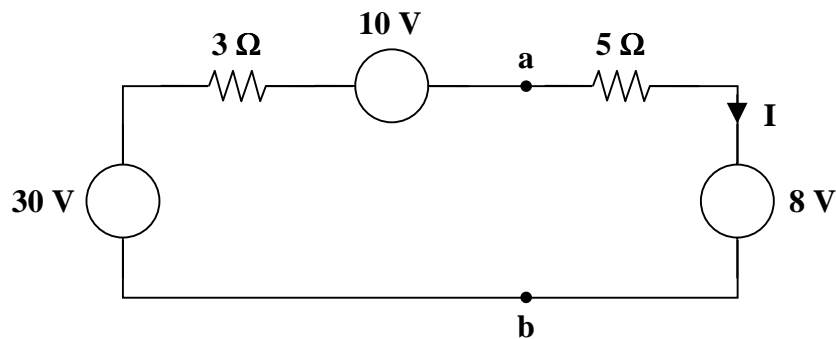$$8 = 1 + 2 + 4 + 1$$
$$8 = 8$$
   This is a valid equation. Thus, our check for accuracy was successful.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
   This problem has been solved satisfactorily.

$$v = \textbf{20 volts}$$

$$i_1 = \textbf{1 amp} \qquad\qquad i_2 = \textbf{2 amps} \qquad\qquad i_3 = \textbf{4 amps} \qquad\qquad i_4 = \textbf{1 amp}$$

Problem 2.8   *[2.15]*   Find I and V_{ab} in the circuit of Figure 2.1.

Figure 2.1

Applying KVL to the loop,
$$-30 + 3I - 10 + 5I + 8 = 0$$
$$8I = 32$$
$$I = \textbf{4 amps}$$

So,
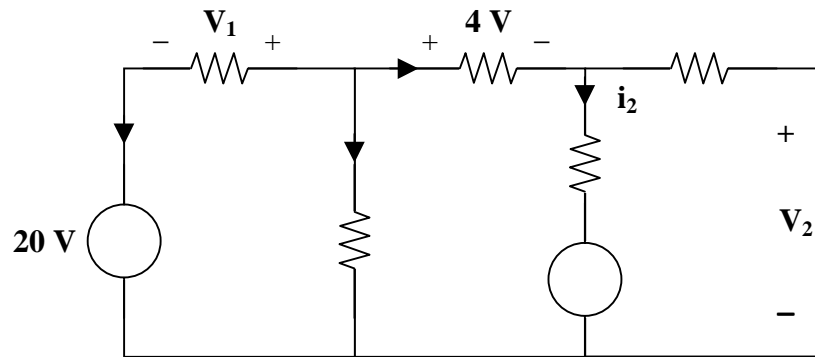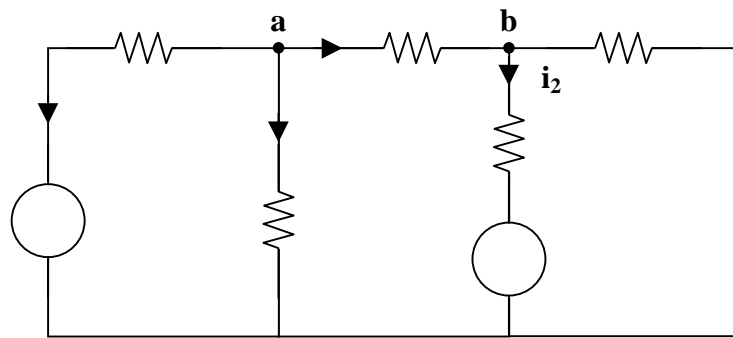$$-V_{ab} + 5I + 8 = 0$$
$$V_{ab} = 5I + 8 = (5)(4) + 8$$

Therefore,
$$V_{ab} = \textbf{28 volts}$$

Problem 2.9   In Figure 2.1, solve for i_1 and i_2.

Figure 2.1

KCL (node a) :   KCL (node b) :
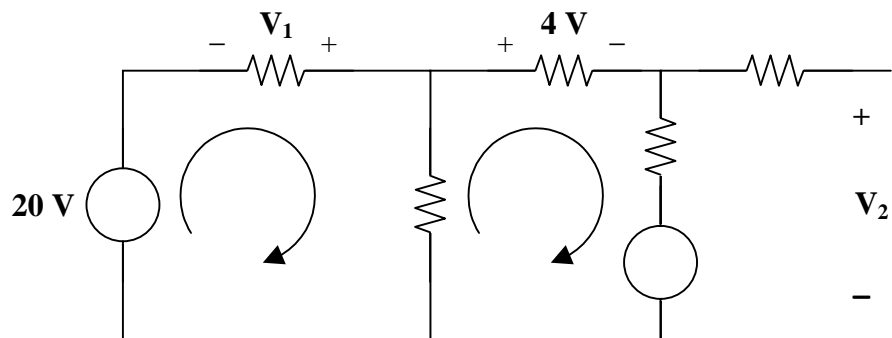
$$i_1 + 3 + 2 = 0$$ $$i_2 + 0 = 3$$

$$i_1 = \textbf{-5 amps}$$ $$i_2 = \textbf{3 amps}$$

Problem 2.10   In Figure 2.1, find V_1 and V_2.

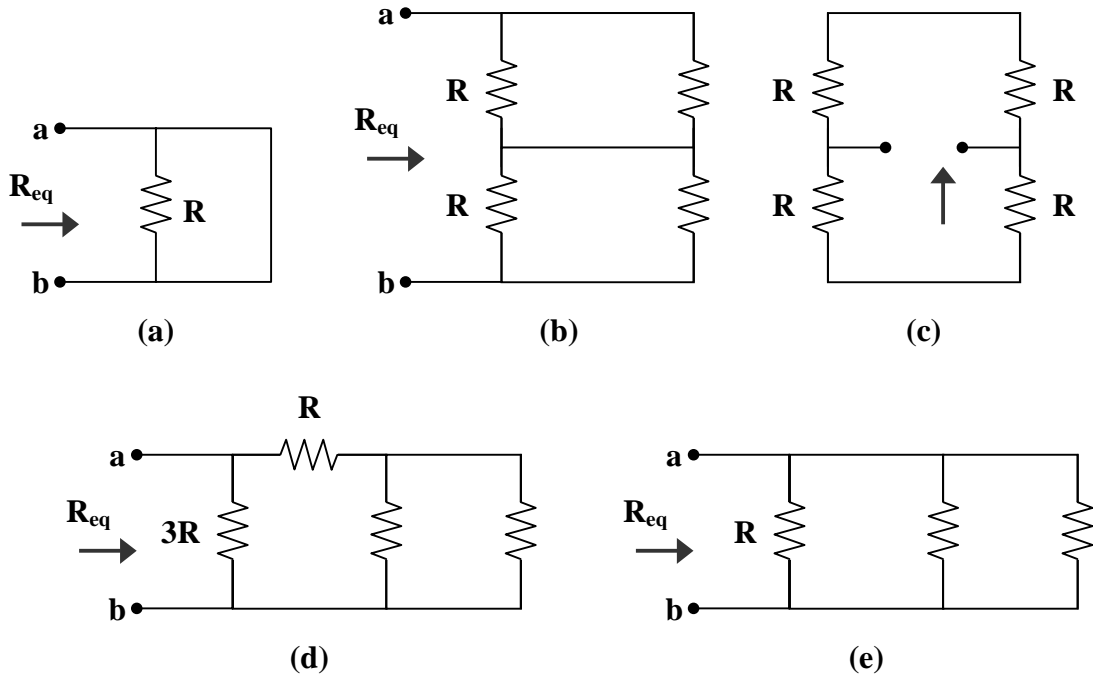$$V_1 = \textbf{-8 volts}$$ $$V_2 = \textbf{8 volts}$$

SERIES AND PARALLEL RESISTORS

Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current. Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Problem 2.11 *[2.35]* Find the equivalent resistance at terminals a-b for each of the following networks.

(a)

(b)

(c)

(d)

(e)

(a) $R_{eq} = R \parallel 0 = 0$

(b) $R_{eq} = R \parallel R + R \parallel R = \dfrac{R}{2} + \dfrac{R}{2} = R$

(c) $R_{eq} = (R + R) \parallel (R + R) = 2R \parallel 2R = R$

(d) $R_{eq} = 3R \parallel (R + R \parallel R) = 3R \parallel \left(R + \dfrac{R}{2}\right) = 3R \parallel \dfrac{3}{2}R = \dfrac{(3R)\left(\dfrac{3}{2}R\right)}{3R + \dfrac{3}{2}R} = \dfrac{9R^2}{9R} = R$
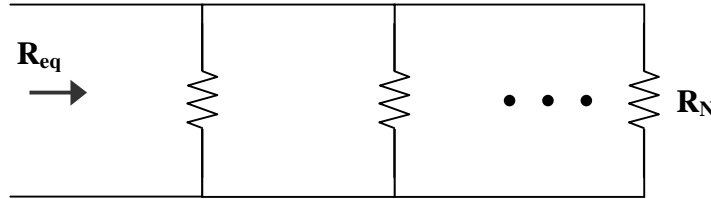
(e) $R_{eq} = R \parallel 2R \parallel 3R = \left(\dfrac{(R)(2R)}{R + 2R}\right) \parallel 3R = \dfrac{2}{3}R \parallel 3R = \dfrac{\left(\dfrac{2}{3}R\right)(3R)}{\dfrac{2}{3}R + 3R} = \dfrac{6R^2}{11R} = \dfrac{6}{11}R$
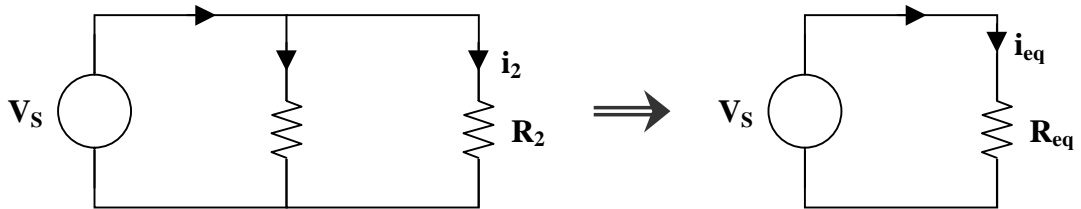
Therefore,

(a) $R_{eq} = \underline{\mathbf{0}}$ (b), (c), (d) $R_{eq} = \underline{\mathbf{R}}$ and (e) $R_{eq} = \dfrac{6}{11}\underline{R}$

Problem 2.12   Given N resistors in parallel, determine if they can be replaced by a single resistor.

Figure 2.1

To find the equivalent resistance of the parallel resistors in Figure 2.1 combine the resistors two at a time to discover a pattern. First, consider the circuits below,

Note that two resistors in parallel have the same voltage across them. Ohm's law gives us
$$V_S = i_1 R_1 = i_2 R_2 \qquad\qquad\qquad V_S = i_{eq} R_{eq}$$

or
$$i_1 = \frac{V_S}{R_1} \qquad \text{and} \qquad i_2 = \frac{V_S}{R_2} \qquad\qquad i_{eq} = \frac{V_S}{R_{eq}}$$

Using KCL,
$$i = i_1 + i_2 \qquad\qquad\qquad\qquad i = i_{eq}$$

Use the equations for i_1 and i_2 and i_{eq} to find i in terms of the source voltage and resistors.
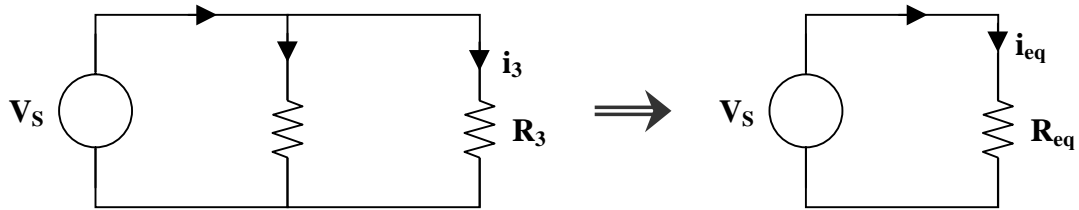$$i = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad\qquad i = \frac{V_S}{R_{eq}}$$

Thus, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ or $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$

This implies that the equivalent resistance of two resistors in parallel is the product of their resistances divided by the sum of their resistances.

Continue the process. Find the equivalent resistance of $R_{1\|2} = \dfrac{R_1 R_2}{R_1 + R_2}$ in parallel with R_3.

Note that two resistors in parallel have the same voltage across them. Ohm's law gives us

$$V_S = i_{1\|2} R_{1\|2} = i_3 R_3 \qquad\qquad\qquad V_S = i_{eq} R_{eq}$$

$$i_{1\|2} = \frac{V_S}{R_{1\|2}} \quad \text{and} \quad i_3 = \frac{V_S}{R_3} \qquad\qquad\qquad i_{eq} = \frac{V_S}{R_{eq}}$$

Using KCL,

$$i = i_{1\|2} + i_3 \qquad\qquad\qquad\qquad\qquad i = i_{eq}$$

Use the equations for $i_{1\|2}$ and i_3 and i_{eq} to find i in terms of the source voltage and resistors.

$$i = \frac{V_S}{R_{1\|2}} + \frac{V_S}{R_3} \qquad\qquad\qquad\qquad i = \frac{V_S}{R_{eq}}$$

$$i = V_S\left(\frac{1}{R_{1\|2}} + \frac{1}{R_3}\right) = V_S\left(\frac{R_1 + R_2}{R_1 R_2} + \frac{1}{R_3}\right) = V_S\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$
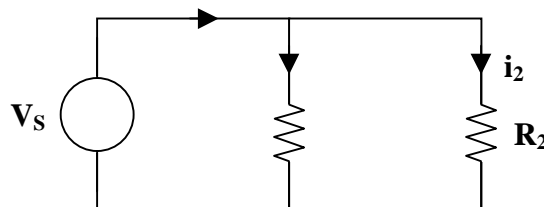
Thus,
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Continuing this process for N resistors would show that the reciprocal of the equivalent resistance of N resistors in parallel is the sum of the reciprocals of each resistance. In general,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

Problem 2.13 If you know the current through two resistors in parallel (R_1 and R_2), is there a simple way to determine the current through either R_1 or R_2?

Clearly, the two resistors in parallel have the same voltage across them. Using Ohm's law,

$$V_S = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{V_S}{R_1} \qquad \text{and} \qquad i_2 = \frac{V_S}{R_2}$$

Using KCL,

$$i = i_1 + i_2$$

Now use the equations for i_1 and i_2 to find i in terms of the source voltage and resistors.

$$i = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_S\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$
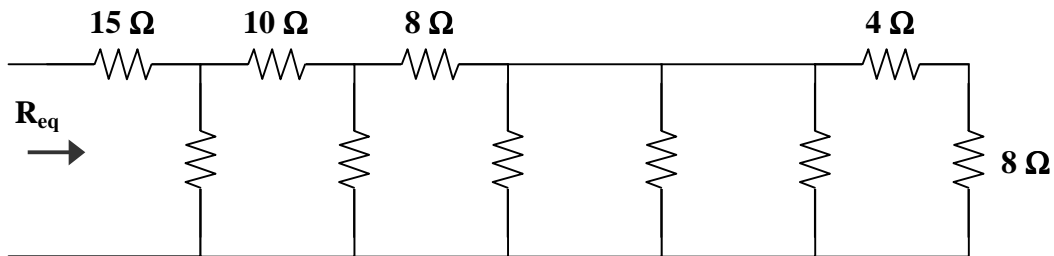
Then,
$$V_S = \left(\frac{R_1 R_2}{R_1 + R_2}\right)i$$

To find the branch currents, substitute the equation for V_S into the equations for i_1 and i_2.

$$i_1 = \frac{1}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)i \qquad\qquad i_2 = \frac{1}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)i$$

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right)i \qquad\qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right)i$$

Thus, it is clear that the current entering the node where two resistors are connected in parallel divides proportionately between the two resistors. The proportionality is equal to the value of the opposite resistor divided by the sum of the resistances times the incoming current. It should be noted that this current division property only works for two resistors in parallel. If you have more than two, you need to use a different process to find how the currents divide.
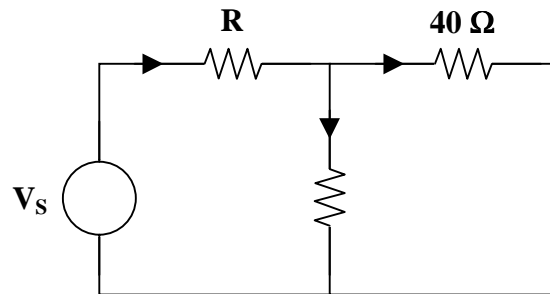

Problem 2.14 Determine R_{eq} for Figure 2.1.

Figure 2.1

$$R_{eq} = \underline{\textbf{25 ohms}}$$

Problem 2.15   Using current division, determine i_1 and i_2 in Figure 2.1.

Figure 2.1

$i_1 =$ **3 amps** $i_2 =$ **4 amps**

WYE-DELTA TRANSFORMATIONS

The following is a summary of the conversions between wye and delta connected loads.

Given the following resistor network, the Y-Δ equations are listed in the left column and the Δ-Y equations are listed in the right column.

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \qquad R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

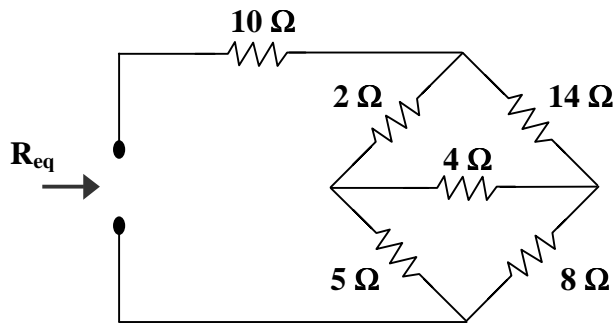**Problem 2.16**   Find R_{eq} for Figure 2.1.

Figure 2.1

➢ **Carefully DEFINE the problem.**
Each resistor has a value and the equivalent resistance is shown to be the resistance of the network at the dotted terminals.

➢ **PRESENT everything you know about the problem.**
To find equivalent resistance, the resistor network must be reduced using series combinations, parallel combinations, and conversions between wye and delta connected resistors. We know that series resistances are added to obtain the equivalent resistance and the inverse of parallel resistances are added to obtain the inverse of the equivalent resistance; i.e.

$$R_{eq} = R_1 + R_2 \qquad \text{and} \qquad 1/R_{eq} = 1/R_1 + 1/R_2$$

We also know how to convert between wye and delta connected loads, as seen previously in this section.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
The set of alternatives to reducing resistor networks involves the various ways that resistors can be combined. In this case, we will convert the lower Δ connection to a Y connection. Then, we will combine parallel resistors (two series resistors are in parallel with two series resistors) to get a series combination. This will produce the equivalent resistance of the resistor network.
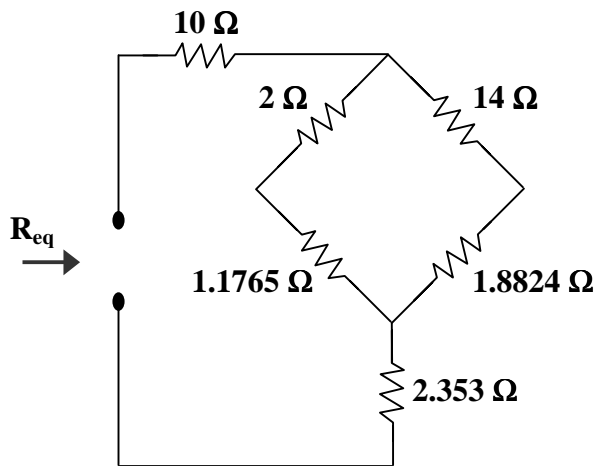
➢ **ATTEMPT a problem solution.**

$$R_a + R_b + R_c = 8 + 5 + 4 = 17$$

$$R_1 = \frac{(4)(5)}{17} = 1.1765$$

$$R_2 = \frac{(4)(8)}{17} = 1.8824$$
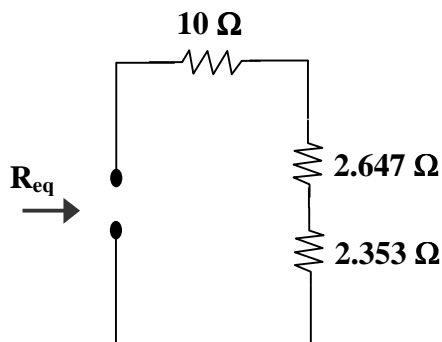
$$R_3 = \frac{(5)(8)}{17} = 2.353$$

$$2 + 1.1765 = 3.177$$
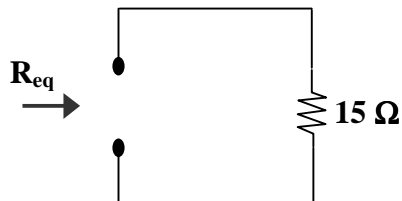$$14 + 1.8824 = 15.882$$

$$3.177 \parallel 15.882$$
$$= \frac{(3.177)(15.882)}{3.177 + 15.882}$$
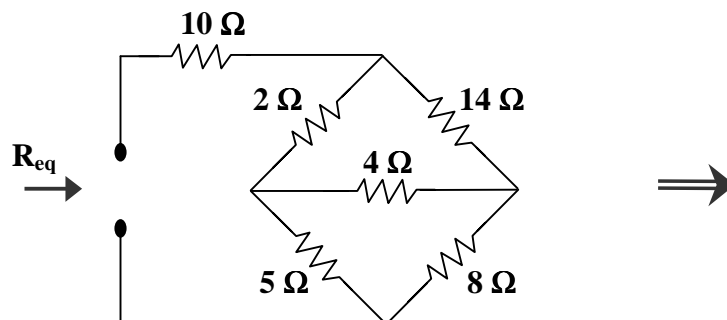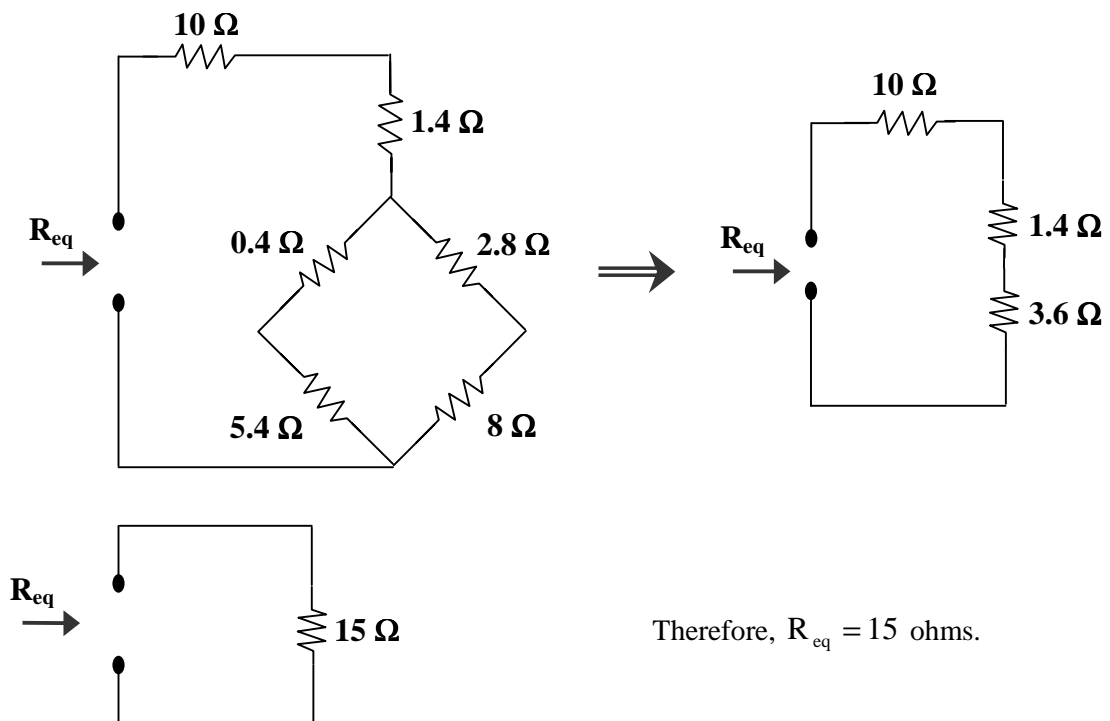$$= \frac{50.46}{19.059} = 2.647$$

$$10 + 2.647 + 2.353 = 15$$

Therefore, $R_{eq} = 15$ ohms.

➤ **EVALUATE the solution and check for accuracy.**
To check for accuracy, reduce the resistor network by converting the upper Δ connection to a Y connection. Then, combine parallel resistors (two series resistors are in parallel with two series resistors) to get a series combination. This will produce the equivalent resistance of the resistor network.
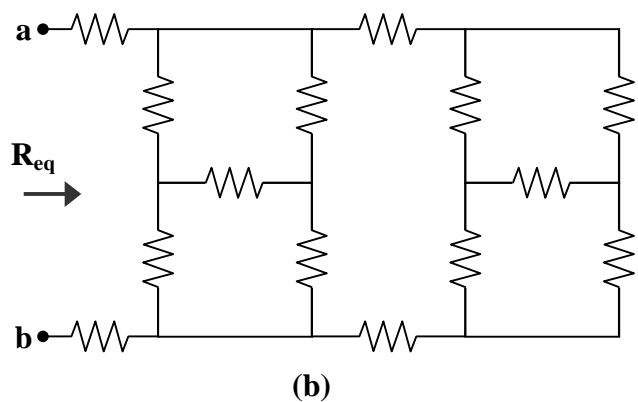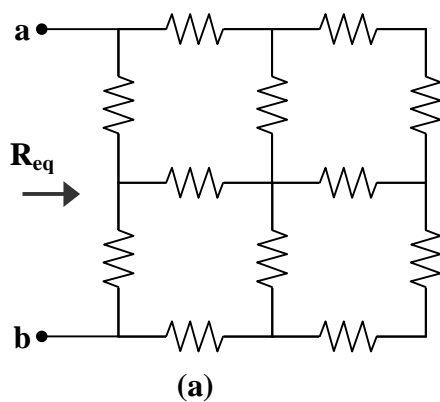
It can be shown that

This is the same answer as that obtained above. Our check for accuracy was successful.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
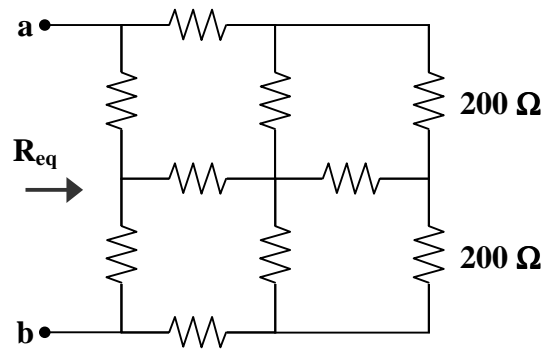This problem has been solved satisfactorily.

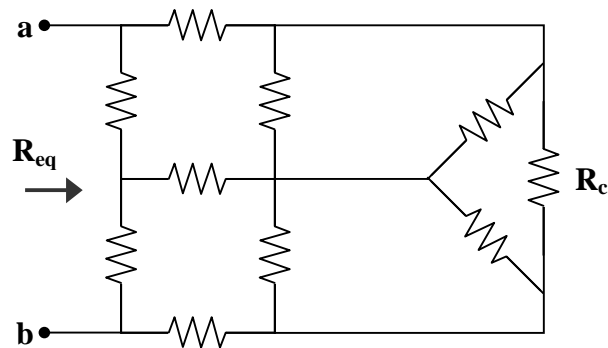$$R_{eq} = \underline{\textbf{15 ohms}}$$

Problem 2.17   *[2.47]*   Find the equivalent resistance R_{eq} in each of the following circuits where each resistor has a value of 100 Ω.

(a)   **(b)**

(a)   Begin by combining the series resistors on the rightmost corners of the network.

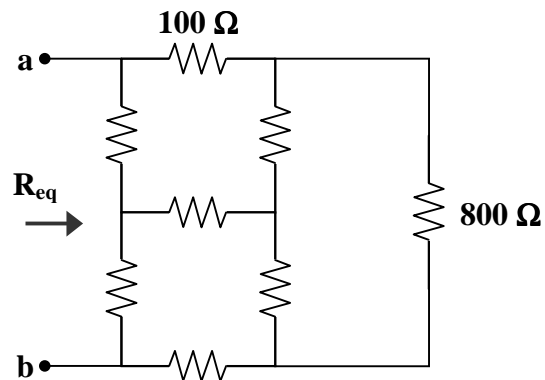Now, convert the T (or Y) connection to a Δ connection.

$$R_a = \frac{(200)(200) + (200)(100) + (100)(200)}{200} = \frac{80000}{200} = 400 \text{ ohms}$$
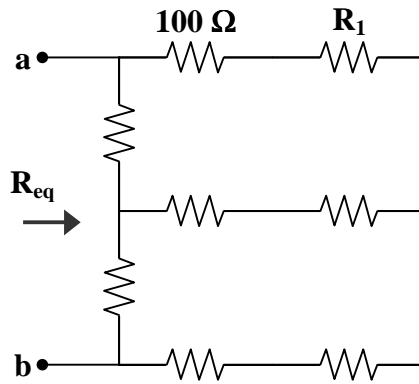
$$R_b = \frac{80000}{200} = 400 \text{ ohms}$$

$$R_c = \frac{80000}{100} = 800 \text{ ohms}$$

Notice that $R_a \parallel 100$ and $R_b \parallel 100$ where $400 \parallel 100 = \frac{(400)(100)}{400 + 100} = \frac{40000}{500} = 80$ ohms.

Now, convert Δ connection to a Y connection.
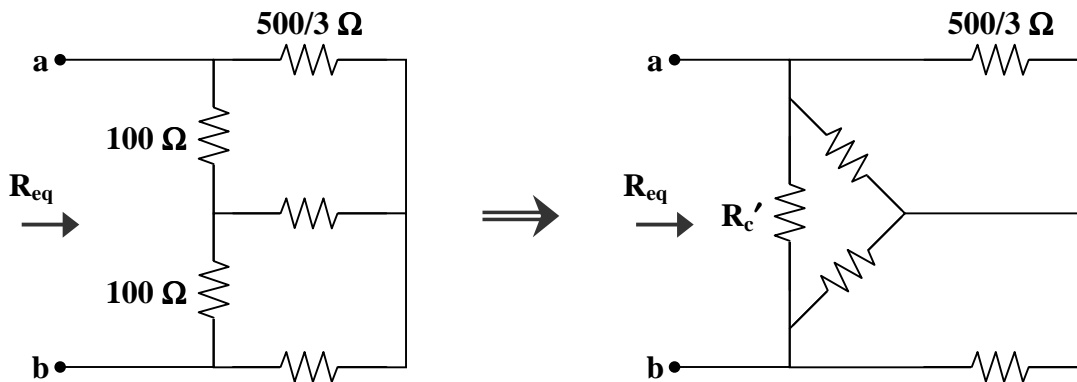

$$R_1 = \frac{(80)(800)}{80+80+800} = \frac{64000}{960} = \frac{200}{3} \text{ ohms}$$

$$R_2 = \frac{(80)(800)}{80+80+800} = \frac{64000}{960} = \frac{200}{3} \text{ ohms}$$

$$R_3 = \frac{(80)(80)}{80+80+800} = \frac{6400}{960} = \frac{20}{3} \text{ ohms}$$

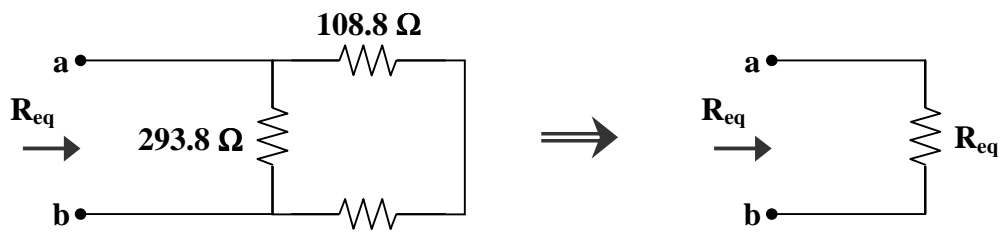Now, combine the series resistors. Then, convert the T (or Y) connection to a Δ connection.

$$(100)(100) + (100)(320/3) + (320/3)(100) = 94000/3$$

$$R_a' = \frac{94000/3}{100} = \frac{940}{3} = 313.3 \text{ ohms}$$

$$R_b' = \frac{94000/3}{100} = \frac{940}{3} = 313.3 \text{ ohms}$$

$$R_c' = \frac{94000/3}{320/3} = \frac{94000}{320} = 293.8 \text{ ohms}$$

Note that $R_a'\|500/3$ and $R_b'\|500/3$ where $\dfrac{(940/3)(500/3)}{(940/3)+(500/3)} = \dfrac{(940)(500)}{(3)(1440)} = 108.8 \text{ ohms.}$
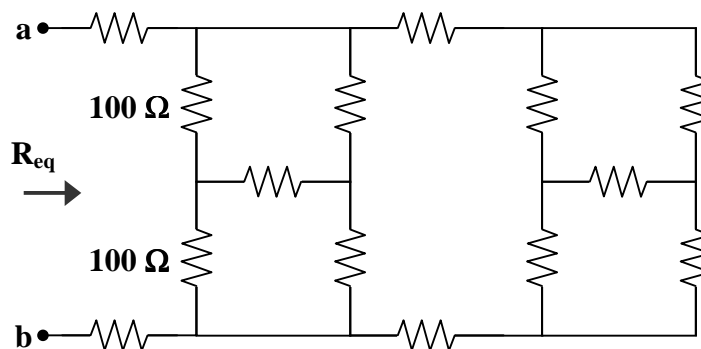
Therefore,
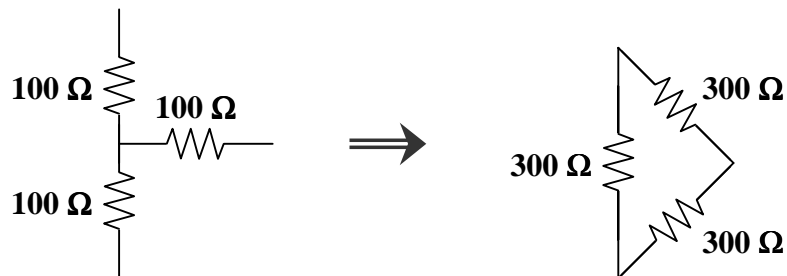
$$R_{eq} = 293.8 \| (108.8 + 108.8) = 293.8 \| 217.6 = \frac{(293.8)(217.6)}{293.8 + 217.6} = \frac{63930}{511.4}$$
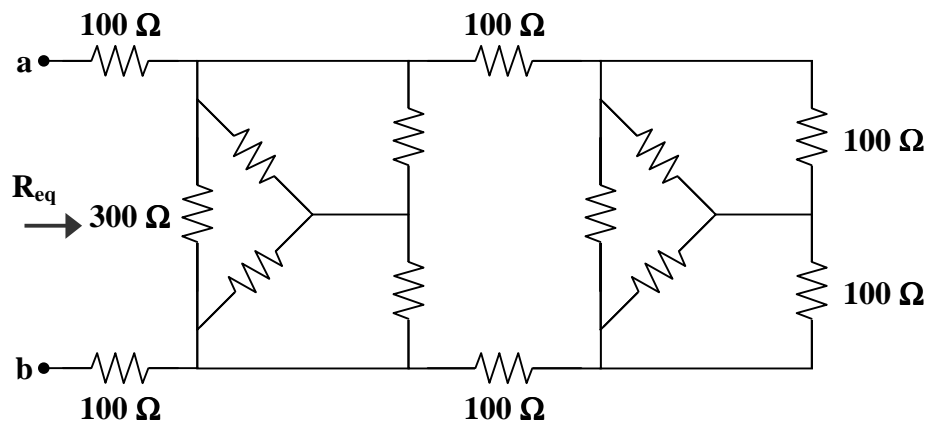
$$R_{eq} = \underline{\textbf{125.01 ohms}}$$

(b)   Convert the T connected resistors, labeled in the circuit below, to Δ connected resistors.
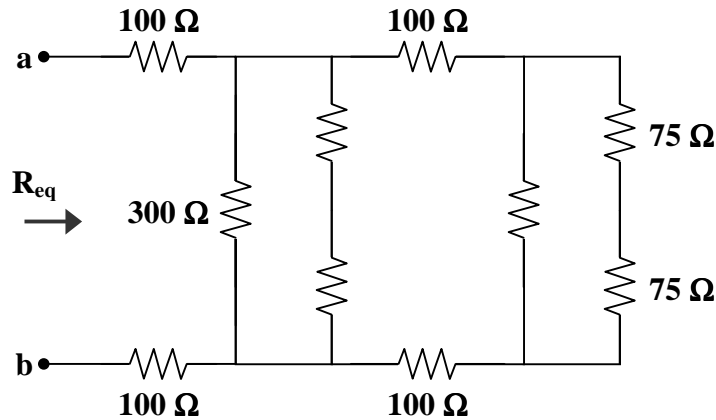

It can be shown that

So, the circuit becomes

Now, each of the slanted branches of the Δ connection is in parallel with a 100 Ω resistor.

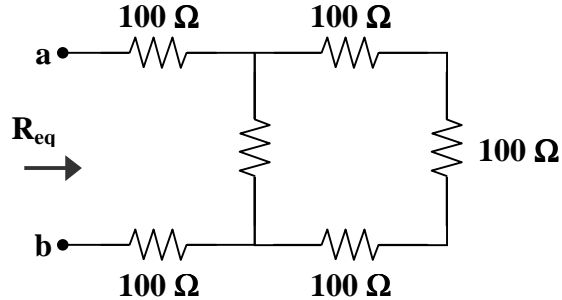$$300 \parallel 100 = \frac{(300)(100)}{300+100} = \frac{30000}{400} = 75 \text{ ohms}$$

This leads to the following circuit.

100 Ω   **100 Ω**

a —/\/\/— ——/\/\/——

75 Ω

$\mathbf{R_{eq}}$ →   **300 Ω**

75 Ω

b —/\/\/—   —/\/\/—
100 Ω   **100 Ω**

Clearly, now we have two places in the circuit where a 300 ohm resistor is in parallel with the series combination of two 75 ohm resistors.

$$300 \parallel (75+75) = 300 \parallel 150 = \frac{(300)(150)}{300+150} = \frac{45000}{450} = 100 \text{ ohms}$$

This simplifies the circuit to

100 Ω   **100 Ω**

a —/\/\/— ——/\/\/——

$\mathbf{R_{eq}}$ →   **100 Ω**

b —/\/\/—   —/\/\/—
100 Ω   **100 Ω**

It is evident that we have

$$R_{eq} = 100 + 100 \parallel (100+100+100) + 100$$

or

$$R_{eq} = 100 + 100 \parallel 300 + 100$$

where we have already shown that $100 \parallel 300 = 75$.

Therefore,

$$R_{eq} = 100 + 75 + 100$$
$$R_{eq} = \underline{\textbf{275 ohms}}$$

APPLICATIONS

Problem 2.18   Given a real voltmeter with an internal resistance of R_m, determine its effect when measuring the voltage across R_2 in Figure 2.1.
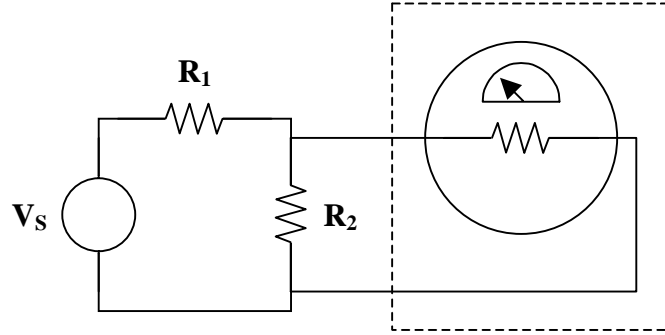

Figure 2.1

➢ **Carefully DEFINE the problem.**
  Each component is labeled, indicating value and polarity. The problem is clear.

➢ **PRESENT everything you know about the problem.**
  The voltmeter measures the voltage across a component and is therefore connected in parallel with the component. As shown in the dashed box in Figure 2.1, the voltmeter consists of a d'Arsonval movement in series with a resistor whose internal resistance is deliberately made very large to minimize the current drawn from the circuit. R_m represents the total resistance of the meter.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  To show the effects of the internal resistance of a real voltmeter,

  (a)   find V_{R2} if R_m is an open circuit, and

  (b)   find V_{R2} if R_m is a short circuit.

➢ **ATTEMPT a problem solution.**

Using voltage division,

$$V_{R2} = \left[\frac{R_2 \| R_m}{R_1 + (R_2 \| R_m)} \right] V_S$$

(a) $\displaystyle \lim_{R_m \to \infty} V_{R2} = \lim_{R_m \to \infty} \left[\frac{R_2 \| R_m}{R_1 + R_2 \| R_m} \right] V_S = \lim_{R_m \to \infty} \left[\frac{(\frac{1}{R_2} + \frac{1}{R_m})^{-1}}{R_1 + (\frac{1}{R_2} + \frac{1}{R_m})^{-1}} \right] V_S = \left[\frac{R_2}{R_1 + R_2} \right] V_S$

(b) $$\lim_{R_m \to 0} V_{R2} = \lim_{R_m \to 0}\left[\frac{R_2 \| R_m}{R_1 + R_2 \| R_m}\right]V_S = \lim_{R_m \to 0}\left[\frac{(\frac{R_2 R_m}{R_2 + R_m})}{R_1 + (\frac{R_2 R_m}{R_2 + R_m})}\right]V_S = \left[\frac{0}{R_1 + 0}\right]V_S = 0$$

➢ **EVALUATE the solution and check for accuracy.**

(a)   If $R_m = \infty$ (open circuit), then all the current flowing through R_1 will continue through R_2. Clearly, using voltage division, $V_{R2} = \left[R_2 / (R_1 + R_2)\right]V_S$

(b)   If $R_m = 0$ (short circuit), then no current will flow through R_2 and $V_{R2} = 0$. This makes sense due to the fact that current flows through the path of least resistance.

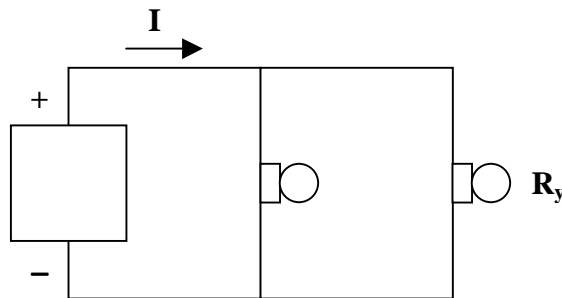➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

If R$_m$ >> R$_2$, the internal resistance will only slightly effect the circuit. As the value of R$_m$ approaches R$_2$, the effect of the internal resistance becomes increasingly more significant.

Problem 2.19   *[2.55]*   As a design engineer, you are asked to design a lighting system consisting of a 70 W power supply and two light bulbs as shown in Figure 2.1. You must select the two bulbs of the following three available bulbs.

(a) $R_1 = 80$ ohms   cost = $0.60 (standard size)

(b) $R_2 = 90$ ohms   cost = $0.90 (standard size)

(c) $R_3 = 100$ ohms   cost = $0.75 (nonstandard size)

The system should be designed for a minimum cost such that $I = 1.2 \text{ A} \pm 5\%$.

Figure 2.1

Since we need two of the three bulbs, there are only three possibilities.

(a)   Use R_1 and R_2 .

$R = R_1 \| R_2 = 80 \| 90 = 42.35$ ohms

$I = 1.2 \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14$ amps

$p = I^2 R = \begin{cases}(1.26)^2(42.35) = 67.23 \text{ W} \\ (1.14)^2(42.35) = 55.04 \text{ W}\end{cases}$

$\cos t = \$0.60 + \$0.90 = \$1.50$

(b)   Use R_1 and R_3 .

$$R = R_1 \| R_3 = 80 \| 100 = 44.44 \text{ ohms}$$

$$I = 1.2 \ \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14 \text{ amps}$$

$$p = I^2 R = \begin{cases} (1.26)^2 (44.44) = 70.55 \text{ W} \\ (1.14)^2 (44.44) = 57.75 \text{ W} \end{cases}$$

$$\cos t = \$0.60 + \$0.75 = \$1.35$$

(c)   Use R_2 and R_3.

$$R = R_2 \| R_3 = 90 \| 100 = 47.37 \text{ ohms}$$

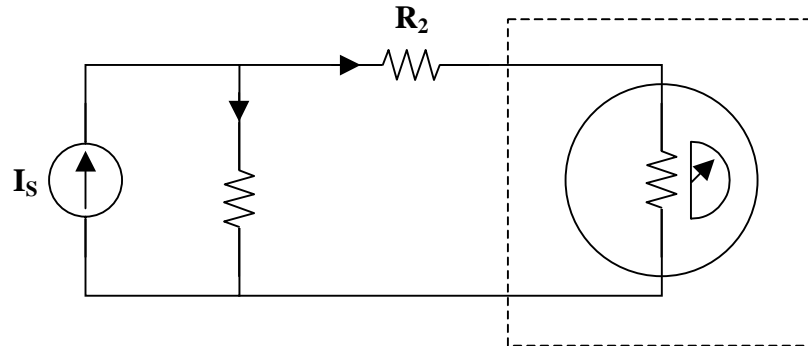$$I = 1.2 \ \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14 \text{ amps}$$

$$p = I^2 R = \begin{cases} (1.26)^2 (47.37) = 75.2 \text{ W} \\ (1.14)^2 (47.37) = 61.56 \text{ W} \end{cases}$$

$$\cos t = \$0.90 + \$0.75 = \$1.65$$

Note that case (b) represents the lowest cost, however both (b) and (c) have a power that exceeds the 70 W power that can be supplied. Therefore, the correct design uses case (a), i.e.

$$\underline{\textbf{R}_1 \text{ and } \textbf{R}_2}.$$

Problem 2.20   Given the circuit in Figure 2.1,

Figure 2.1

find I_{R2} for $I_S = 10$ mA, $R_{sh} = 5 \ \Omega$, and

(a) $R_1 = 10 \ k\Omega$ and $R_2 = 10 \ k\Omega$

(b) $R_1 = 1000 \ \Omega$ and $R_2 = 1000 \ \Omega$

(c) $R_1 = 50 \ \Omega$ and $R_2 = 50 \ \Omega$

(d) $R_1 = 5 \ \Omega$ and $R_2 = 5 \ \Omega$

Using current division,

$$I_{R2} = \left[\frac{R_1}{R_1 + (R_2 + R_{sh})} \right] I_S$$

(a) $I_{R2} = \left[\dfrac{10 \times 10^3}{(10 \times 10^3) + (10 \times 10^3) + 5} \right] (10 \times 10^{-3}) = \left[\dfrac{100}{2.001 \times 10^4} \right] = 4.998 \text{ mA}$

(b) $I_{R2} = \left[\dfrac{1 \times 10^3}{(1 \times 10^3) + (1 \times 10^3) + 5} \right] (10 \times 10^{-3}) = \left[\dfrac{10}{2.005 \times 10^3} \right] = 4.987 \text{ mA}$

(c) $I_{R2} = \left[\dfrac{50}{50 + 50 + 5} \right] (10 \times 10^{-3}) = \left[\dfrac{0.5}{105} \right] = 4.762 \text{ mA}$

(d) $I_{R2} = \left[\dfrac{5}{5 + 5 + 5} \right] (10 \times 10^{-3}) = \left[\dfrac{0.05}{15} \right] = 3.333 \text{ mA}$

In summary,
(a) $I_{R2} =$ **4.998 mA**
(b) $I_{R2} =$ **4.987 mA**
(c) $I_{R2} =$ **4.762 mA**
(d) $I_{R2} =$ **3.333 mA**

These answers can be compared to the case where $R_{sh} = 0$ to see just how much the current through R_2 is affected by the internal resistance of the real ammeter. In each case, it can be shown that

$$I_{R2} = \left[\frac{R_1}{R_1 + R_2} \right] I_S = (0.5)(10 \times 10^{-3}) = 5 \text{ mA}$$